

## Optimum Truncation of a Gaussian Beam for Propagation Through Atmospheric Turbulence

15 November 1994

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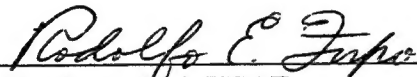


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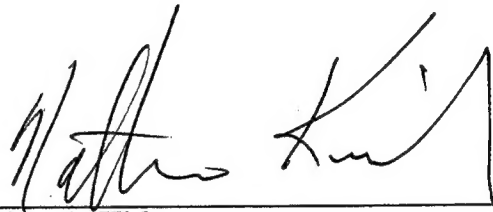
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## Contents

1. Introduction.....	1
2. Far-Field Distribution in the Absence of Turbulence .....	3
3. Mean On-Axis Far-Field Irradiance in the Presence of Turbulence.....	7
4. Optimum Truncation.....	11
5. Discussion.....	15
6. Conclusions.....	17
References.....	19
Appendix.....	21

## Figures

1. The mean on-axis irradiance, $\langle I(0) \rangle I_u$ for uncompensated turbulence as a function of the ratio of the beam diameter in the aperture to the truncation diameter for various values of $D/r_0$ .....	11
2. The optimum ratio of the beam diameter to the truncation diameter as a function of $D/r_0$ for uncompensated turbulence .....	12
3. The maximum mean on-axis irradiance, $\langle I(0) \rangle I_u$ for uncompensated turbulence as a function of $D/r_0$ .....	13
4. A comparison of the mean on-axis irradiance, $\langle I(0) \rangle I_u$ , obtained from optimum truncation of a Gaussian beam (solid curve) and from that obtained if the same total power is uniformly distributed over the aperture (dashed curve).....	15
A-1. A comparison of the exact (solid curve) and the approximate (dashed curve) Strehl ratio for uncompensated turbulence of an infinite Gaussian-shaped initial optical field.....	22

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## 1. Introduction

In many optical systems employing laser beams that propagate through the atmosphere, it is desirable, for many applications, to obtain maximum far-field (or focal-plane) irradiance, for a given output laser power, on or near the optic axis. Examples of such systems include: optical radars, where one wants to obtain the highest possible signal return; optical communication systems, where one wants to operate under conditions of the highest possible signal-to-noise ratio; and laser systems, where one wants to deliver the highest possible power density to a target. In these systems (and others) one is often concerned with the far-field (or focal-plane) irradiance pattern that is within the central lobe of the transmitted beam, the corresponding much weaker side lobes being of less relative importance. Since the outputs of many laser systems are best approximated by a truncated Gaussian optical field across a circular aperture, a quantitative understanding of the manner in which such a beam propagates through atmospheric turbulence is required for the prediction of the performance of various systems employing lasers. Previous efforts along these lines have required extensive numerical computations<sup>1-3</sup> because no analytic results have been available. Here, we develop elementary analytical approximations for the propagation of a truncated Gaussian beam through atmospheric turbulence.

It is assumed that the primary degradation of beam propagation is due to clear air atmospheric turbulence. One of the major degrading effects of atmospheric turbulence on an optical beam is turbulence-induced beam spread and the corresponding reduction of the average irradiance at some (distant) observation plane.<sup>4</sup> We consider uncompensated optical systems and derive expressions for the optimum ratio of the beam diameter (in the transmitting aperture) to the physical aperture diameter that maximizes the corresponding far-field irradiance. Based on this optimum ratio, we then obtain corresponding results for the mean irradiance distribution and maximum mean on-axis irradiance. It is well known that if a focusing lens is placed directly in front of an aperture the angular irradiance pattern in the focal plane is identical to that in the far-field. Thus, the results derived below apply equally as well to the focal plane. As such, the results of this paper will be useful to systems analysts involved with the design of optical systems that operate in the presence of turbulence where adaptive optics turbulence corrections are either impractical or not available for implementation.

In Sec. 2, we present an accurate analytic approximation to the far-field irradiance distribution of a truncated Gaussian beam in the absence of turbulence that is valid within the central lobe of the exact distribution. Based on this approximation, in Sec. 3, we derive a corresponding analytic approximation for the mean irradiance distribution in the presence of turbulence. In Sec. 4, we maximize the mean on-axis irradiance as a function of the ratio of the beam diameter to the truncation diameter and obtain the corresponding maximum mean on-axis far-field irradiance. Previous analysis of the propagation of truncated Gaussian beams through turbulence have been numerical in nature. Thus, the accurate elementary approximations obtained here allow analytic results to be obtained for the optimum mean irradiance distribution in the presence of turbulence, which, in many cases, facilitate the parametric estimation and optimization of overall system performance.

## 2. Far-Field Distribution in the Absence of Turbulence

We consider an unobscured aperture of diameter  $D$  that truncates a ( $\text{TEM}_{00}$ ) Gaussian-shaped laser beam of  $1/e^2$  intensity radius  $\omega_0$  and incident power  $P$ . In the absence of turbulence, and assuming that the (multiplicative) transmission coefficient due to molecular and particulates equals unity, the irradiance distribution of wavelength  $\lambda$  and propagation range  $z$  in the far field can be written as

$$I(\theta) = I(0) G(\theta), \quad (2.1)$$

where  $I(0)$ , the on-axis irradiance, is given by

$$I(0) = I_u F_{\text{trunc}}, \quad (2.2)$$

$$I_u = \frac{PA}{(\lambda z)^2}, \quad (2.3)$$

$$F_{\text{trunc}} = \frac{2[1 - \exp(-\mu^2)]^2}{\mu^2}. \quad (2.4)$$

$A$  is the area of the aperture ( $= \pi D^2/4$ ), and  $G(\theta)$ , the beam pattern, is given by

$$G(\theta) = \left[ \frac{2 \int_0^\mu dx x \exp(-x^2) J_0(ux/\mu)}{1 - \exp(-\mu^2)} \right]^2, \quad (2.5)$$

where  $J_0$  is the Bessel function of the first kind of order zero,

$$u = \frac{\theta}{(\lambda/\pi D)}, \quad (2.6)$$

$$\mu = \frac{D}{d}, \quad (2.7)$$

and  $d = 2\omega_0$  is the  $1/e^2$  intensity beam diameter in the plane of the aperture. In Eq. (2.2), the quantity  $I_0$  is the on-axis far-field irradiance that is obtained for uniform illumination of the aperture (which is the absolute maximum irradiance obtained under any physical circumstances), and  $F_{\text{trunc}} (< 1)$  gives the effects of truncation of the Gaussian beam by the aperture. Increasing the beam diameter to better fill the aperture will decrease the far-field angular pattern, but beyond a certain point will also cause increasing power loss as the beam is truncated by the aperture. For an aperture of fixed diameter  $D$ , it is well known<sup>5</sup> that the maximum on-axis far-field irradiance is obtained for  $d \approx 0.89D$ . For this value of  $d$ ,  $F_{\text{trunc}} \approx 0.81$ . Thus, in the absence of turbulence, the maximum far-field on-axis irradiance is about 81% of what would be obtained if the same total power is uniformly distributed over the circular aperture; this condition occurs for  $d \approx 0.89D$ .

As is shown below, the optimum value of  $d/D$  in the presence of turbulence is obtained for  $d/D < 0.89$ . For this case, it has been shown<sup>6</sup> that an accurate analytical approximation to the far-field irradiance distribution,  $I_A$ , which is valid within the main central lobe, is given by

$$I_A(\theta) = I(0)G_A(\theta), \quad (2.8)$$

where  $I(0)$ , the on axis far-field irradiance, is given by Eq. (2.2), and

$$G_A(\theta) = \exp\left[-2\theta^2 / \theta_0^2\right], \quad (2.9)$$

where the  $1/e^2$  intensity beam half-width angle,  $\theta_0$ , is given by

$$\theta_0 = \sqrt{\left(\frac{2\lambda}{\pi d}\right)^2 + \left(\frac{2\lambda b}{\pi D}\right)^2}, \quad (2.10)$$

where  $b$ , determined from the requirement that the total integrated far-field irradiance of  $I_A$  equals the exact value of the transmitted power, is given by

$$b = \frac{\sqrt{2}\mu}{\sqrt{\exp(\mu^2) - 1}}. \quad (2.11)$$

The approximate irradiance distribution,  $I_A$ , gives both the exact on-axis irradiance and total integrated irradiance. In addition, the encircled power distributions are valid to better than about 2%. Over the range of interest (i.e.,  $d/D < 1$ ), the difference between the approximate and the exact distributions are well within the uncertainties that always exist in practice.

Note, the characteristic  $1/e^2$  beam half-width angle,  $\theta_0$ , can be written as

$$\theta_0 = \frac{2\lambda}{\pi D_{\text{eff}}}, \quad (2.12)$$

where

$$D_{\text{eff}} = d \sqrt{\tanh(D^2 / 2d^2)}. \quad (2.13)$$

The quantity  $D_{\text{eff}}$  is the  $1/e^2$  beam intensity diameter in the aperture plane of an “effectively” infinite Gaussian field that results in the distribution  $I_A$ . That is, we replace the original Gaussian field of  $1/e^2$  diameter  $d$  and the aperture of diameter  $D$  by an infinite Gaussian field of  $1/e^2$  diameter  $D_{\text{eff}}$ , which, when propagated to the far-field, results in an irradiance distribution  $I_A$  that is a very good approximation to what would be obtained from the truncated Gaussian. We note that  $I_A$  does not describe correctly the side lobes and should not be used when the corresponding distribution is important. It is a valid approximation within the main central lobe only. In the next section, we use this infinite Gaussian field to obtain the corresponding results for the mean irradiance distribution in the presence of turbulence.



### 3. Mean On-Axis Far-Field Irradiance in the Presence of Turbulence

In the presence of uncompensated atmospheric turbulence, the far-field mean irradiance distribution,  $\langle I(\theta) \rangle$ , can be written as<sup>7</sup> (angular brackets denote the ensemble average)

$$\langle I(\theta) \rangle = I(0)SR, \quad (3.1)$$

In Eq. (2.1),  $I(0)$ , the on-axis irradiance in the absence of turbulence for arbitrary illumination of an aperture, is given by

$$I(0) = I_u F_{\text{trunc}}, \quad (3.2)$$

where  $I_u$ , the on-axis irradiance for uniform illumination of the aperture, is given by Eq. (2.3),  $F_{\text{trunc}}$ , the effect of aperture truncation, is given by

$$F_{\text{trunc}} = \frac{\left| \int_{\text{ap}} U_o(\mathbf{r}) d^2 \mathbf{r} \right|^2}{PA}. \quad (3.3)$$

Additionally,  $SR$ , the uncompensated turbulence Strehl ratio, which gives the reduction in on-axis mean irradiance due to turbulence, is given by

$$SR = \int_{\text{ap}} K(\mathbf{r}) \exp\left(-\frac{1}{2} D_w(\mathbf{r})\right) d^2 \mathbf{r}, \quad (3.4)$$

where

$$K(\mathbf{r}) = \frac{\int_{\text{ap}} U_o\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right) U_o^*\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) d^2 \mathbf{R}}{\left| \int_{\text{ap}} U_o(\mathbf{R}) d^2 \mathbf{R} \right|^2}, \quad (3.5)$$

$U_o$  is the (arbitrary) complex optical field in the aperture plane. The subscript "ap" on the integrals indicates an integration over the aperture, and  $D_w$  is the wave-structure function, given, for the Kolmogorov spectrum in the inertial subrange, by<sup>8</sup>

$$D_w(\mathbf{r}) = 2 \left( \frac{r}{\rho_0} \right)^{5/3}, \quad (3.6)$$

where the lateral coherence length,  $\rho_o$ , is given by

$$\rho_o = \left[ \frac{2.91}{2} k^2 \int_0^z ds (s/z)^{5/3} C_n^2(s) \right]^{-3/5}, \quad (3.7)$$

where  $k$  is the optical wave number ( $2\pi/\lambda$ ), and  $C_n^2(s)$  is the index structure constant profile along the propagation path. In Eq.(3.5), the origin of integration is located in the observation plane. In many cases, the SR ( $\leq 1$ ) is used as figure of merit. Values of  $SR \ll 1$  indicates poor performance, while an SR near unity indicates performance close to that obtained in the absence of turbulence.

Although Eqs. (3.1)–(3.7) apply for arbitrary  $U_o$ , we are interested here in the case where  $U_o$  is of Gaussian shape. In particular, as discussed in Sec.2, for determination of the irradiance distribution of a truncated Gaussian beam, we can replace the truncated Gaussian and the circular aperture and propagate the infinite Gaussian initial optical field given by

$$U_o(r) = C \exp \left[ -\frac{r^2}{(D_{\text{eff}}/2)^2} \right] \quad (0 \leq r \leq \infty), \quad (3.8)$$

where  $C$  is a constant, and  $D_{\text{eff}}$  is given in Eq. (2.13). As discussed in Sec. 2, this initial optical field leads to accurate analytical approximations for both the far-field irradiance and encircled power distributions of a truncated Gaussian beam. When the infinite Gaussian wave function given in Eq. (3.8) is substituted in Eq. (3.4), it can be shown (see Appendix) that an accurate analytical engineering expression for the uncompensated turbulence Strehl ratio is given by

$$SR \approx \frac{1}{1 + 2 \left( \frac{D_{\text{eff}}}{r_o} \right)^2}, \quad (3.9)$$

where  $r_o = (3.44)^{3/5}$ , and  $r_o$  is called the Fried coherence length in the literature.<sup>9</sup> Typically, near  $0.5 \mu\text{m}$ ,  $r_o$  is about 5–10 cm for vertical propagation through the atmosphere. Below, we use the dimensionless parameter  $D/r_o$  to characterize the effects of turbulence, where increasing values of  $D/r_o$  corresponds to increasing turbulence degradations.

For completeness, we note that the turbulence-induced mean irradiance distribution,  $G_T(\theta)$ , can be obtained by inserting the multiplicative factor  $\exp(ik\theta \cdot \mathbf{r})$  in the integral in the numerator of Eq. (3.4). The corresponding approximate mean irradiance distribution,  $G_{AT}(\theta)$ , which is based on Eq. (3.8), is given by

$$G_{AT}(\theta) \approx SR \exp\left(-\frac{2\theta^2}{\theta_0^2 + \theta_T^2}\right), \quad (3.10)$$

where  $\theta_0$  is the angular beam spread in the absence of turbulence, given by Eq. (2.10), and  $\theta_T$ , the turbulence induced angular beam spread, is given by

$$\theta_T = \frac{2\sqrt{2}\lambda}{\pi r_0}, \quad (3.11)$$

where SR is the Strehl ratio given in Eq. (3.9). The irradiance distribution obtained from Eq. (3.10) gives both the correct mean on-axis and total integrated irradiance (i.e., equal to the power,  $P_T$ , transmitted through the aperture:  $P_T = \{1 - \exp[-2\mu^2]\}P$ ).

#### 4. Optimum Truncation

Substituting Eqs. (2.4) and (3.9) into Eq. (3.1) yields

$$\langle I(0) \rangle = I_u \left( \frac{2[1 - \exp(-\mu^2)]^2}{\mu^2} \right) \left( \frac{1}{1 + \frac{2(D/r_0)^2}{b^2 + \mu^2}} \right), \quad (4.1)$$

where  $b$  is given by Eq. (2.11), and  $\mu = D/d$ . The quantity in the first bracket on the right hand side of Eq. (4.1) (i.e.,  $F_{\text{trunc}}$ ) gives the effects of truncation, and the corresponding second quantity (i.e.,  $SR$ ) gives the effects of turbulence on reducing the on-axis irradiance with respect to that of uniform illumination of the aperture.

Now,  $F_{\text{trunc}}$  reaches its maximum for  $d = 0.89D$ . On the other hand,  $SR$  is a decreasing function of  $d$ . In particular,  $SR(d < 0.89D) > SR(d > 0.89D)$ . Hence, it follows that the value of  $d$  that maximizes the mean on-axis irradiance will be obtained for  $d < 0.89D$ . This property is shown in Figure 1 where  $\langle I(0) \rangle / I_u$  is plotted as a function of  $d/D$  for various values of  $D/r_0$ . Examination of Figure 1 reveals, for  $D/r_0$  less than about 2, that the on-axis irradiance has a relatively sharp peak as a function of  $d/D$ , and the effects of optimum truncation are rather pronounced. On the other hand, for values of  $D/r_0$  greater than about 3, the corresponding irradiance has a rather broad peak, and the effects of optimum truncation are correspondingly less significant. This is as expected since the effects of uncompensated turbulence produce an irradiance pattern with less detailed structure than the corresponding pattern in the absence of turbulence and with the maxima and minima tending to be averaged out.

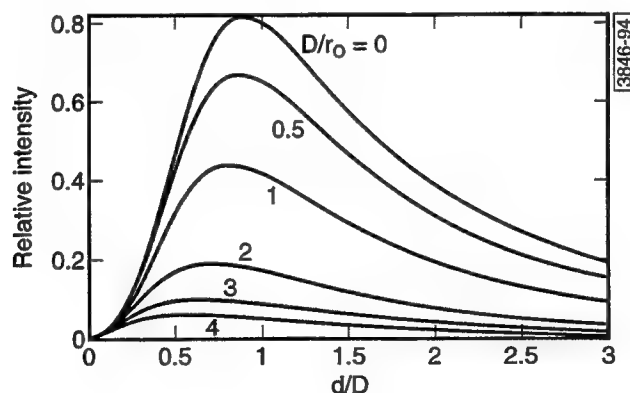


Figure 1. The mean on-axis irradiance,  $\langle I(0) \rangle / I_u$ , for uncompensated turbulence as a function of the ratio of the beam diameter in the aperture to the truncation diameter for various values of  $D/r_0$ .

Based on Eq. (4.1), we can readily obtain results for the optimum truncation ratio of  $d/D$  that yields the maximum mean on-axis far-field irradiance. Differentiating Eq. (4.1) with respect to “ $d$ ” and setting the result equal to zero yields a transcendental equation for  $(d/D)_{\text{opt}}$ :

$$2\mu_o^2 - (e^{\mu_o^2} - 1) \left( 1 - \frac{4(D/r_o)^2}{(e^{\mu_o^2} + 1)^2} \right) = 0, \quad (4.2)$$

where  $\mu_o = (D/d)_{\text{opt}}$ . Although this equation can not be solved analytically, numerical results for  $(d/D)_{\text{opt}}$  are readily obtained and are plotted in Figure 2 as a function of  $D/r_o$ . An accurate analytical approximation for  $(d/D)_{\text{opt}}$  for  $0 \leq D/r_o \leq 10$ , as obtained by a least-squares fit to the calculated values, is given by

$$(d/D)_{\text{opt}} = \begin{cases} 0.892 - 0.0536(D/r_o)^2 + 0.0089(D/r_o)^4, & \text{for } 0 \leq (D/r_o) \leq 1.25 \\ 0.468 + \frac{0.592}{\sqrt{D/r_o}} - \frac{0.085}{\sqrt{\ln D/r_o}}, & \text{for } 1.25 < (D/r_o) \leq 10 \end{cases} \quad (4.3)$$

The accuracy of this approximation is better than 1% over the range  $0 \leq D/r_o \leq 10$ .

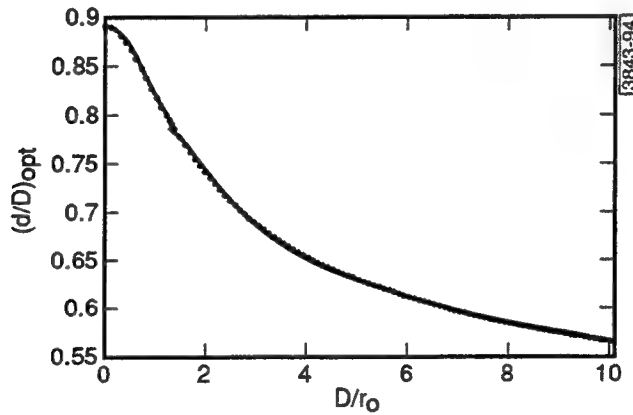


Figure 2. The optimum ratio of the beam diameter to the truncation diameter as a function of  $D/r_o$  for uncompensated turbulence. The dotted and solid curves are obtained from the calculated numerical values and Eq. (4.2), respectively.

Of course, for  $D/r_0 = 0$  (i.e., in the absence of turbulence), the results shown in Figures 1 and 2 are identical to the results given in Chap. 18 of Ref. 2. On the other hand, it can be shown directly from the equation that determines  $(d/D)_{\text{opt}}$  that for  $D/r_0 \gg 1$ ,  $(d/D)_{\text{opt}} \approx 1/\sqrt{\ln(D/r_0)}$ , which decreases very slowly as a function of  $D/r_0$ . For example, for  $D/r_0 = 1$  and 2, examination of Figure 2 reveals that  $(d/D)_{\text{opt}} = 0.82$  and 0.74, respectively. Additionally, Figure 3 is a plot of the maximum mean on-axis irradiance as a function of  $D/r_0$  for uncompensated turbulence. For the examples given above, examination of Figure 3 shows that the corresponding maximum mean irradiances are about 44 and 19%, respectively, of what would be obtained for uniform illumination of the aperture in the absence of turbulence.

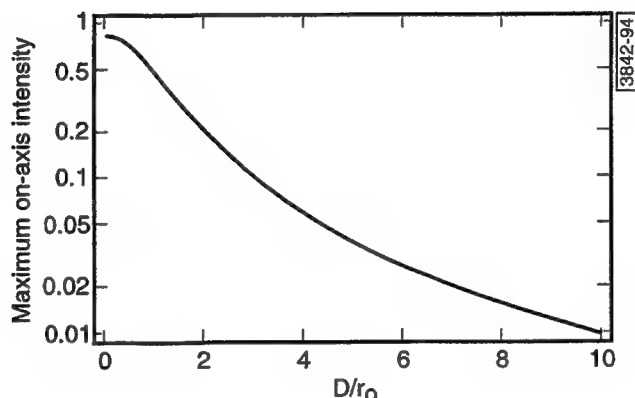


Figure 3. The maximum mean on-axis irradiance,  $\langle I(0) \rangle I_u$ , for uncompensated turbulence as a function of  $D/r_0$ . For each value of  $D/r_0$ , the optimum value of  $d/D$  has been used to obtain the maximum mean on-axis irradiance.

## 5. Discussion

Figure 2 shows that the optimum beam diameter,  $d_{\text{opt}}$ , is a monotonically decreasing function of  $D/r_0$ . For  $D/r_0 \gg 1$ , examination of Eqs. (2.11) and (2.13) reveals that  $D_{\text{eff}} \approx d_{\text{opt}}$ . As a result, the corresponding optimum Strehl ratio is an explicit function of  $d_{\text{opt}}/r_0$ , which approaches unity with decreasing values of  $d_{\text{opt}}$ . Hence, in this limit, optimum truncation in the presence of turbulence is a trade off between  $d \rightarrow 0$ , where the  $\text{SR} \rightarrow 1$  (i.e., the effects of turbulence are negligible as  $d \rightarrow 0$ ), and  $d \rightarrow 0.89D$ , where  $F_{\text{trunc}}$  reaches its maxima (i.e., where truncation effects are minimal).

Next, we compare the results of optimum Gaussian beam truncation to the corresponding results obtained if the same total power in the beam is uniformly distributed over the aperture. The primary range of interest is for  $D/r_0 > 1$  because otherwise the effects of turbulence are rather benign. As a result, we use approximate analytical results below for the Strehl ratios for uniform illumination, which are highly accurate for  $D/r_0 > 1$ . Note, for uniform illumination,  $F_{\text{trunc}} = 1$ , and  $\langle I_u(0) \rangle / I_u = \text{SR}_u$ , where  $\text{SR}_u$  is the corresponding Strehl ratio for uniform illumination.

$$\text{SR}_u \approx \frac{1}{1 + (D/r_0)^2}. \quad (4.4)$$

Figure 4 shows a comparison of the mean on-axis irradiance of the optimally truncated Gaussian,  $\langle I(0) \rangle / I_u = (F_{\text{trunc}} \text{SR})_{\text{opt}}$ , with the corresponding mean on-axis irradiance obtained for uniform illumination,  $\langle I_u(0) \rangle / I_u = \text{SR}_u$ . Examination of Figure 4 reveals that, for all practical purposes, for  $D/r_0 > 1$ , optimum truncation of a Gaussian beam and uniform illumination result in the same on-axis irradiance in the presence of uncompensated turbulence.

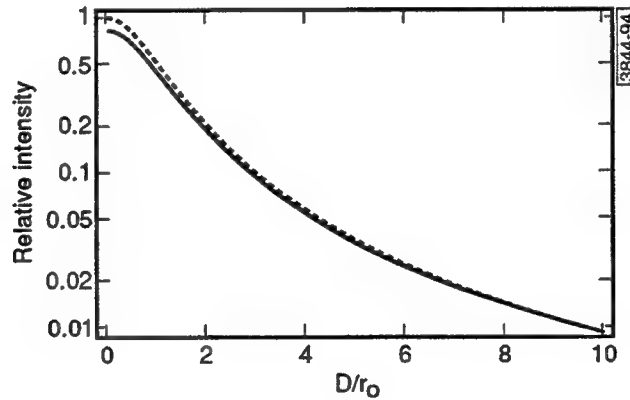


Figure 4. A comparison of the mean on-axis irradiance,  $\langle I(0) \rangle / I_u$ , obtained from optimum truncation of a Gaussian beam (solid curve) and from that obtained if the same total power is uniformly distributed over the aperture (dashed curve). In both cases, uncompensated turbulence is assumed.

## 6. Conclusions

Based on elementary analytical approximations to the far-field irradiance distribution of a truncated Gaussian in the absence of turbulence, we have derived expressions both for the turbulence-induced mean far-field (or focal-plane) irradiance pattern and corresponding Strehl ratio. The maximum mean on-axis irradiance has then been obtained, as well as the corresponding optimum ratio of the beam diameter to the aperture diameter as a function of the strength of turbulence. For uncompensated turbulence and moderate turbulence strength (i.e., for  $D/r_0 < 3$ ), the mean on-axis irradiance has a rather pronounced peak about  $d/D_{\text{opt}}$ , and the results obtained here will have useful applications to uncompensated laser systems that must operate in this regime (e.g., for systems that operate near  $10\text{ }\mu\text{m}$ ). For all practical purposes, for  $D/r_0 > 1$ , optimum truncation of a Gaussian beam and uniform illumination of a circular aperture of diameter  $D$  result in the same on-axis irradiance in the presence of uncompensated turbulence.



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## Appendix

In this Appendix, we derive the far-field (or focal-plane) Strehl ratio for an infinite Gaussian optical field. Consider the infinite Gaussian optical field given by

$$U(r) = U_0 \exp\left(-\frac{r^2}{a_g^2}\right) \quad (0 \leq r \leq \infty), \quad (\text{A-1})$$

where  $a_g$  is the  $1/e^2$  intensity radius of the beam in the initial plane. Substituting Eq. (A-1) into Eq. (3.5) and integrating over the entire initial plane yields

$$K(r) = \frac{1}{2\pi a_g^2} \exp\left(-\frac{r^2}{2a_g^2}\right). \quad (\text{A-2})$$

Substituting Eqs. (3.6) and (A-2) into Eq. (3.4) and performing the angular integration yields

$$SR = \left(\frac{\rho_0}{a_g}\right)^2 \int_0^\infty \exp(-x^{5/3}) \exp\left(-\frac{1}{2}x^2(\rho_0/a_g)^2\right) x dx. \quad (\text{A-3})$$

For  $a_g/\rho_0 \ll 1$ , the first factor within the integrand of Eq. (A-3) is small in comparison with the second and may be neglected, and we obtain that

$$SR \approx 1 \quad (a_g/\rho_0 \ll 1). \quad (\text{A-4})$$

On the other hand, for  $a_g/\rho_0 \gg 1$ , the first factor in the integrand of

Eq. (A-3) is dominant, and we obtain

$$\begin{aligned} SR &\approx (\rho_0/a_g)^2 \int_0^\infty \exp(-x^{5/3}) x dx \\ &= \frac{3}{5} \Gamma(6/5) (\rho_0/a_g)^2 \\ &= 0.50 (r_0/D_g)^2 \quad (a_g/\rho_0 \gg 1), \end{aligned} \quad (\text{A-5})$$

where  $\Gamma$  is the Gamma function,  $D_g = 2a_g$ , and  $r_0 = (3.44)^{3/5}\rho_0$ . A convenient engineering approximation for the SR can be then expressed as

$$SR \cong \frac{1}{1 + 2\left(\frac{D_g}{r_0}\right)^2}, \quad (A-6)$$

which is used in Eq. (3.8) with  $D_g = D_{eff}$ . Figure A-1 is a comparison between the exact SR, based on Eq. (A-3), and the approximate SR, based on Eq. (A-6). Examination of Figure 5 reveals that the largest differences of about 7.7% occur near  $D_g/r_0 \approx 0.5$ . This represents very good agreement and justifies its use as an effective engineering approximation.

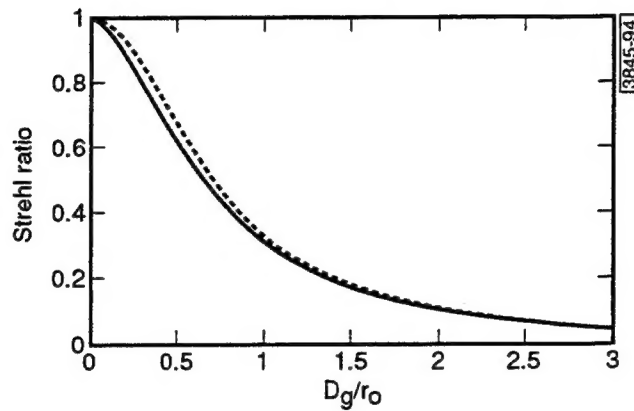


Figure A-1. A comparison of the exact (solid curve) and the approximate (dashed curve) Strehl ratio for uncompensated turbulence of an infinite Gaussian-shaped initial optical field.

## TECHNOLOGY OPERATIONS

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